CALCULUS.

220 (Incorrectly numbered 219). For solution, see page 85.

221 (Incorrectly numbered 220, p. 90. For solution see page 148.

222 (Incorrectly numbered 221, p. 117). Proposed by Professor F. ANDEREGG, Oberlin College, Oberlin, O.

If a, b, c, ... represent all the prime numbers 2, 3, 5, ... prove that

$$(1+\frac{1}{a^2})(1+\frac{1}{b^2})(1+\frac{1}{c^2})\,.....=\frac{15}{\pi^2}.$$

No solution has been received.

223 (Incorrectly numbered 222, page 117). Proposed by REV. R. D. CARMICHAEL, Anniston, Ala.

Evaluate $\int_0^1 (1+x^m)^n \log x\, dx$.

Solution by G. W. GREENWOOD, M. A., Dunbar, Pa.

Assuming that n is integral, we have

$$\int_0^1 (1+x^m)^n \log x dx=\int \sum_{r=0}^{n} c_r x^{mr}\log x dx= \sum \frac{c_r[(mr+1)x^{mr+1}\log x-x^{mr+1}]}{(mr+1)^2}\Bigg]_0^1$$

$$=-\sum\frac{c_r}{(mr+1)^2},\text{ since }\lim_{x\doteq 0} x^n\log x=0.$$

224 (Incorrectly numbered 221, page 153). Proposed by REV. R. D. CARMICHAEL, Anniston, Ala.

Find $\lim_{x\doteq 0}\tan^{-1}x(\log x)$.

Solution by EDWIN L. RICH, Schenectady, N. Y.

$$\lim_{x\doteq 0}\tan^{-1}x\log x=\left.\frac{\log x}{\dfrac{1}{\tan^{-1}x}}\right\}_{x\doteq 0}=\left.\frac{\dfrac{d}{dx}(\log x)}{\dfrac{d}{dx}\left(\dfrac{1}{\tan^{-1}x}\right)}\right\}_{x\doteq 0}$$

$$=\frac{x^2+1}{x}\left(x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\text{ etc.}\right)^2\Bigg]_{x\doteq 0}$$

$$=(x^2+1)(1-\tfrac{1}{3}x^2+\tfrac{1}{5}x^4-\tfrac{1}{7}x^6+\text{ etc.})(x-\tfrac{1}{3}x^3+\tfrac{1}{5}x^5-\tfrac{1}{7}x^7+\text{ etc.})]_{x\doteq 0}=0.$$

Also solved by J. Scheffer and G. W. Greenwood.

225 (Incorrectly numbered 222). Proposed by REV. R. D. CARMICHAEL, Anniston, Ala.

If $s_n=2\,(\frac{1}{n}-\frac{2}{2n^3}+\frac{1}{5n^5}+\frac{1}{7n^7}-\frac{2}{9n^9}+\frac{1}{11n^{11}}+\ldots)$ prove that

$$\log 3=s_3+s_4,$$
$$\log 7=s_2+s_3+s_4,$$
$$\log 13=s_2+2s_3+s_4.$$